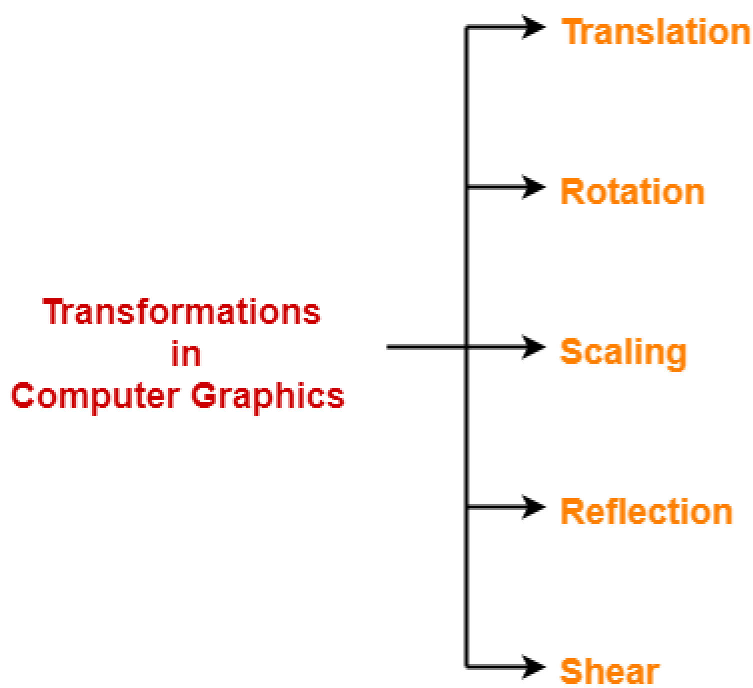
3D Translation in Computer Graphics

In Computer graphics,
Transformation is a process of modifying and re-positioning the existing graphics.

- 3D Transformations take place in a three dimensional plane.
- 3D Transformations are important and a bit more complex than 2D Transformations.
- Transformations are helpful in changing the position, size, orientation, shape etc of the object.

Transformation Techniques-

In computer graphics, various transformation techniques are-



1. Translation
2. Rotation
3. Scaling

In this article, we will discuss about 3D Translation in Computer Graphics.

3D Translation in Computer Graphics-

In Computer graphics,

3D Translation is a process of moving an object from one position to another in a three dimensional plane.

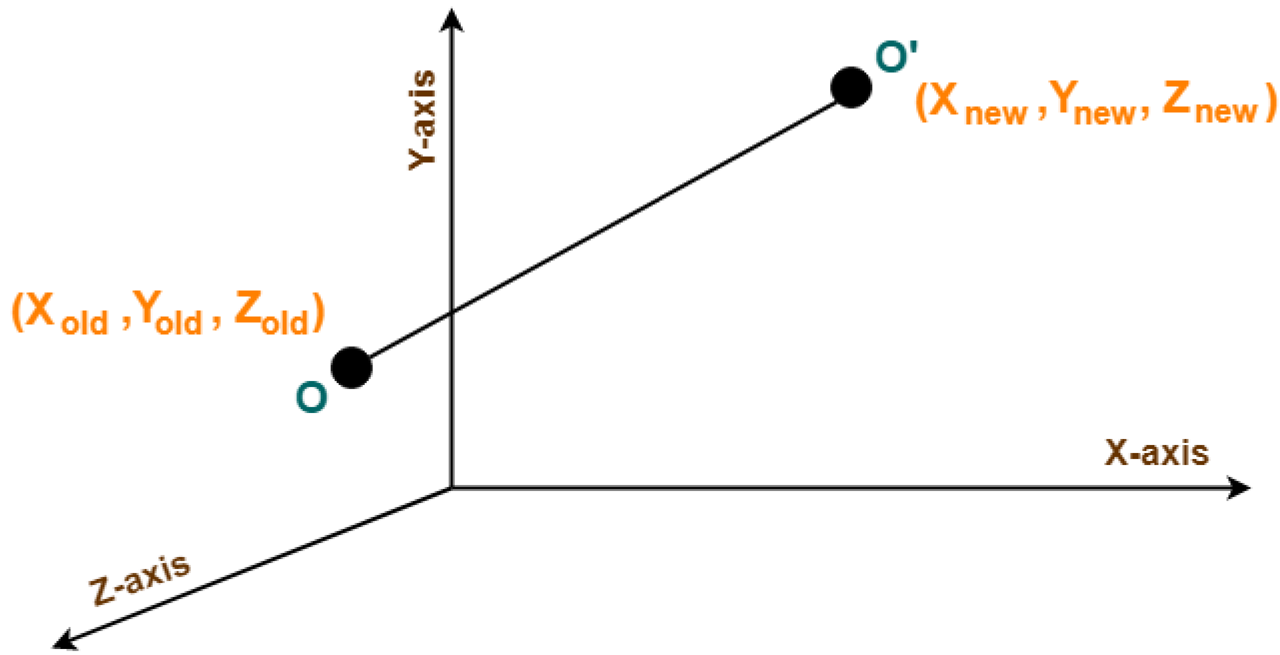
Consider a point object O has to be moved from one position to another in a 3D plane.

Let-

- Initial coordinates of the object O = (X_{old}, Y_{old}, Z_{old})
- New coordinates of the object O after translation = (X_{new}, Y_{new}, Z_{old})
- Translation vector or Shift vector = (T_x, T_y, T_z)

Given a Translation vector (T_x, T_y, T_z)-

- T_x defines the distance the X_{old} coordinate has to be moved.
- T_y defines the distance the Y_{old} coordinate has to be moved.
- T_z defines the distance the Z_{old} coordinate has to be moved.



3D Translation in Computer Graphics

This translation is achieved by adding the translation coordinates to the old coordinates of the object as-

- $X_{new} = X_{old} + T_x$ (This denotes translation towards X axis)
- $Y_{new} = Y_{old} + T_y$ (This denotes translation towards Y axis)
- $Z_{new} = Z_{old} + T_z$ (This denotes translation towards Z axis)

In Matrix form, the above translation equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Translation Matrix

Also Read- 2D Translation in Computer Graphics

PRACTICE PROBLEM BASED ON 3D TRANSLATION IN COMPUTER GRAPHICS-

Problem-

Given a 3D object with coordinate points A(0, 3, 1), B(3, 3, 2), C(3, 0, 0), D(0, 0, 0). Apply the translation with the distance 1 towards X axis, 1 towards Y axis and 2 towards Z axis and obtain the new coordinates of the object.

Solution-

Given-

- Old coordinates of the object = A (0, 3, 1), B(3, 3, 2), C(3, 0, 0), D(0, 0, 0)
- Translation vector = (T_x, T_y, T_z) = (1, 1, 2)

For Coordinates A(0, 3, 1)

Let the new coordinates of A = (X_{new}, Y_{new}, Z_{new}).

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 0 + 1 = 1$
- $Y_{new} = Y_{old} + T_y = 3 + 1 = 4$
- $Z_{new} = Z_{old} + T_z = 1 + 2 = 3$

Thus, New coordinates of A = (1, 4, 3).

For Coordinates B(3, 3, 2)

Let the new coordinates of B = (X_{new}, Y_{new}, Z_{new}).

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 3 + 1 = 4$
- $Y_{new} = Y_{old} + T_y = 3 + 1 = 4$
- $Z_{new} = Z_{old} + T_z = 2 + 2 = 4$

Thus, New coordinates of B = (4, 4, 4).

For Coordinates C(3, 0, 0)

Let the new coordinates of C = (X_{new}, Y_{new}, Z_{new}).

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 3 + 1 = 4$
- $Y_{new} = Y_{old} + T_y = 0 + 1 = 1$
- $Z_{new} = Z_{old} + T_z = 0 + 2 = 2$

Thus, New coordinates of C = (4, 1, 2).

For Coordinates D(0, 0, 0)

Let the new coordinates of D = (X_{new}, Y_{new}, Z_{new}).

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 0 + 1 = 1$
- $Y_{new} = Y_{old} + T_y = 0 + 1 = 1$
- $Z_{new} = Z_{old} + T_z = 0 + 2 = 2$

Thus, New coordinates of D = (1, 1, 2).

Thus, New coordinates of the object = A (1, 4, 3), B(4, 4, 4), C(4, 1, 2), D(1, 1, 2).

3D Rotation in Computer Graphics-

In Computer graphics,

3D Rotation is a process of rotating an object with respect to an angle in a three dimensional plane.

Consider a point object O has to be rotated from one angle to another in a 3D plane.

Let-

- Initial coordinates of the object O = $(X_{old}, Y_{old}, Z_{old})$
- Initial angle of the object O with respect to origin = Φ
- Rotation angle = θ
- New coordinates of the object O after rotation = $(X_{new}, Y_{new}, Z_{new})$

In 3 dimensions, there are 3 possible types of rotation-

- X-axis Rotation
- Y-axis Rotation
- Z-axis Rotation

For X-Axis Rotation-

This rotation is achieved by using the following rotation equations-

- $X_{new} = X_{old}$
- $Y_{new} = Y_{old} \times \cos\theta - Z_{old} \times \sin\theta$
- $Z_{new} = Y_{old} \times \sin\theta + Z_{old} \times \cos\theta$

In Matrix form, the above rotation equations may be represented as-

$$
\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}
$$

3D Rotation Matrix

(For X-Axis Rotation)

For Y-Axis Rotation-

This rotation is achieved by using the following rotation equations-

- $X_{new} = Z_{old} \times \sin\theta + X_{old} \times \cos\theta$
- $Y_{new} = Y_{old}$
- $Z_{new} = Y_{old} \times \cos\theta - X_{old} \times \sin\theta$

In Matrix form, the above rotation equations may be represented as-

$$
\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}
$$

3D Rotation Matrix

(For Y-Axis Rotation)

For Z-Axis Rotation-

This rotation is achieved by using the following rotation equations-

- $X_{new} = X_{old} \times \cos\theta - Y_{old} \times \sin\theta$
- $Y_{new} = X_{old} \times \sin\theta + Y_{old} \times \cos\theta$
- $Z_{new} = Z_{old}$

In Matrix form, the above rotation equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Rotation Matrix

(For Z-Axis Rotation)

PRACTICE PROBLEMS BASED ON 3D ROTATION IN COMPUTER GRAPHICS-

Problem-01:

Given a homogeneous point (1, 2, 3). Apply rotation 90 degree towards X, Y and Z axis and find out the new coordinate points.

Solution-

Given-

- Old coordinates = $(X_{old}, Y_{old}, Z_{old})$ = (1, 2, 3)

- Rotation angle = θ = 90º

For X-Axis Rotation-

Let the new coordinates after rotation = (X_{new}, Y_{new}, Z_{new}).

Applying the rotation equations, we have-

- $X_{new} = X_{old} = 1$
- $Y_{new} = Y_{old} \times \cos\theta - Z_{old} \times \sin\theta = 2 \times \cos 90° - 3 \times \sin 90° = 2 \times 0 - 3 \times 1 = -3$
- $Z_{new} = Y_{old} \times \sin\theta + Z_{old} \times \cos\theta = 2 \times \sin 90° + 3 \times \cos 90° = 2 \times 1 + 3 \times 0 = 2$

Thus, New coordinates after rotation = (1, -3, 2).

For Y-Axis Rotation-

Let the new coordinates after rotation = (X_{new}, Y_{new}, Z_{new}).

Applying the rotation equations, we have-

- $X_{new} = Z_{old} \times \sin\theta + X_{old} \times \cos\theta = 3 \times \sin 90° + 1 \times \cos 90° = 3 \times 1 + 1 \times 0 = 3$
- $Y_{new} = Y_{old} = 2$
- $Z_{new} = Y_{old} \times \cos\theta - X_{old} \times \sin\theta = 2 \times \cos 90° - 1 \times \sin 90° = 2 \times 0 - 1 \times 1 = -1$

Thus, New coordinates after rotation = (3, 2, -1).

For Z-Axis Rotation-

Let the new coordinates after rotation = (X_{new}, Y_{new}, Z_{new}).

Applying the rotation equations, we have-

- $X_{new} = X_{old} \times \cos\theta - Y_{old} \times \sin\theta = 1 \times \cos 90° - 2 \times \sin 90° = 1 \times 0 - 2 \times 1 = -2$
- $Y_{new} = X_{old} \times \sin\theta + Y_{old} \times \cos\theta = 1 \times \sin 90° + 2 \times \cos 90° = 1 \times 1 + 2 \times 0 = 1$
- $Z_{new} = Z_{old} = 3$

Thus, New coordinates after rotation = (-2, 1, 3).

3D Scaling in Computer Graphics-

In computer graphics, scaling is a process of modifying or altering the size of objects.

- Scaling may be used to increase or reduce the size of object.
- Scaling subjects the coordinate points of the original object to change.
- Scaling factor determines whether the object size is to be increased or reduced.
- If scaling factor > 1, then the object size is increased.
- If scaling factor < 1, then the object size is reduced.

Consider a point object O has to be scaled in a 3D plane.

Let-

- Initial coordinates of the object O = (X_{old}, Y_{old},Z_{old})
- Scaling factor for X-axis = S_x
- Scaling factor for Y-axis = S_y
- Scaling factor for Z-axis = S_z
- New coordinates of the object O after scaling = (X_{new}, Y_{new}, Z_{new})

This scaling is achieved by using the following scaling equations-

- $X_{new} = X_{old} \times S_x$
- $Y_{new} = Y_{old} \times S_y$
- $Z_{new} = Z_{old} \times S_z$

In Matrix form, the above scaling equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Scaling Matrix

PRACTICE PROBLEMS BASED ON 3D SCALING IN COMPUTER GRAPHICS-

Problem-01:

Given a 3D object with coordinate points A(0, 3, 3), B(3, 3, 6), C(3, 0, 1), D(0, 0, 0). Apply the scaling parameter 2 towards X axis, 3 towards Y axis and 3 towards Z axis and obtain the new coordinates of the object.

Solution-

Given-

- Old coordinates of the object = A (0, 3, 3), B(3, 3, 6), C(3, 0, 1), D(0, 0, 0)
- Scaling factor along X axis = 2
- Scaling factor along Y axis = 3
- Scaling factor along Z axis = 3

For Coordinates A(0, 3, 3)

Let the new coordinates of A after scaling = (X_{new}, Y_{new}, Z_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 0 \times 2 = 0$
- $Y_{new} = Y_{old} \times S_y = 3 \times 3 = 9$
- $Z_{new} = Z_{old} \times S_z = 3 \times 3 = 9$

Thus, New coordinates of corner A after scaling = (0, 9, 9).

For Coordinates B(3, 3, 6)

Let the new coordinates of B after scaling = (X_{new}, Y_{new}, Z_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 3 \times 2 = 6$
- $Y_{new} = Y_{old} \times S_y = 3 \times 3 = 9$
- $Z_{new} = Z_{old} \times S_z = 6 \times 3 = 18$

Thus, New coordinates of corner B after scaling = (6, 9, 18).

For Coordinates C(3, 0, 1)

Let the new coordinates of C after scaling = (X_{new}, Y_{new}, Z_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 3 \times 2 = 6$
- $Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$
- $Z_{new} = Z_{old} \times S_z = 1 \times 3 = 3$

Thus, New coordinates of corner C after scaling = (6, 0, 3).

For Coordinates D(0, 0, 0)

Let the new coordinates of D after scaling = (X_{new}, Y_{new}, Z_{new}).

Applying the scaling equations, we have-

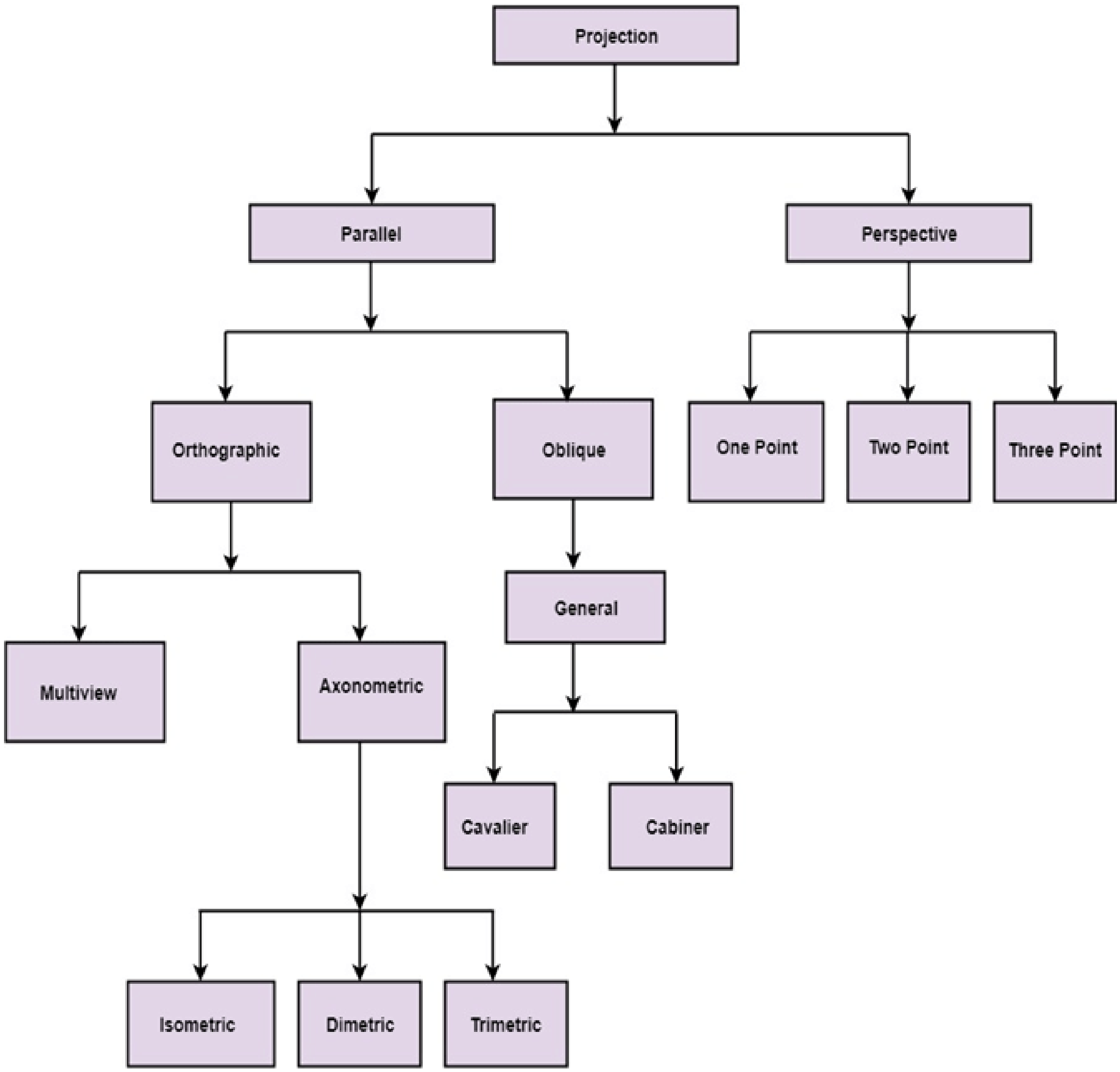
- $X_{new} = X_{old} \times S_x = 0 \times 2 = 0$
- $Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$
- $Z_{new} = Z_{old} \times S_z = 0 \times 3 = 0$

Thus, New coordinates of corner D after scaling = (0, 0, 0).

Projection

It is the process of converting a 3D object into a 2D object. It is also defined as mapping or transformation of the object in projection plane or view plane. The view plane is displayed surface.

Types of Projection



Perspective Projection

In perspective projection farther away object from the viewer, small it appears. This property of projection gives an idea about depth. The artist use perspective projection from drawing three-dimensional scenes.

Two main characteristics of perspective are vanishing points and perspective foreshortening. Due to foreshortening object and lengths appear smaller from the center of projection. More we increase the distance from the center of projection, smaller will be the object appear.

Vanishing Point

It is the point where all lines will appear to meet. There can be one point, two point, and three point perspectives.

One Point: There is only one vanishing point as shown in fig (a)

Two Points: There are two vanishing points. One is the x-direction and other in the y -direction as shown in fig (b)

Three Points: There are three vanishing points. One is x second in y and third in two directions.

In Perspective projection lines of projection do not remain parallel. The lines converge at a single point called a center of projection. The projected image on the screen is obtained by points of intersection of converging lines with the plane of the screen. The image on the screen is seen as of viewer's eye were located at the centre of projection, lines of projection would correspond to path travel by light beam originating from object.

Important terms related to perspective

1. **View plane:** It is an area of world coordinate system which is projected into viewing plane.
2. **Center of Projection:** It is the location of the eye on which projected light rays converge.
3. **Projectors:** It is also called a projection vector. These are rays start from the object scene and are used to create an image of the object on viewing or view plane.

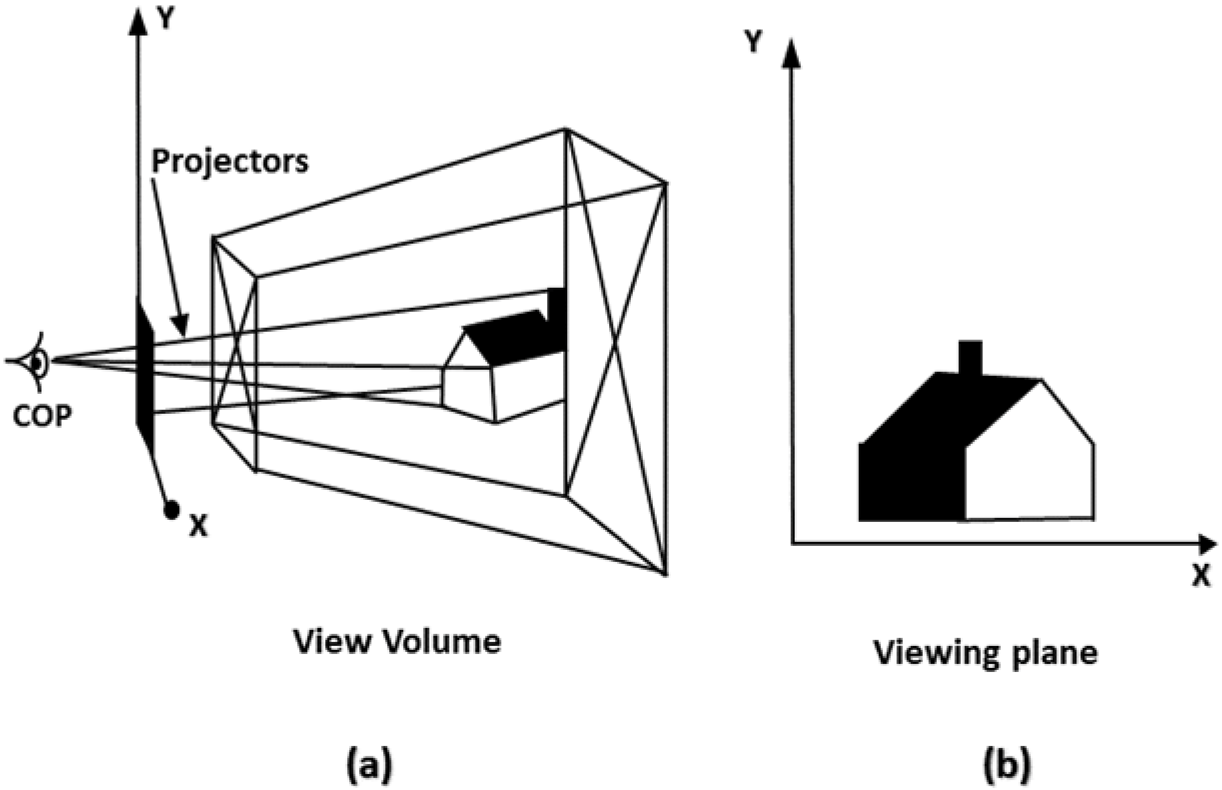
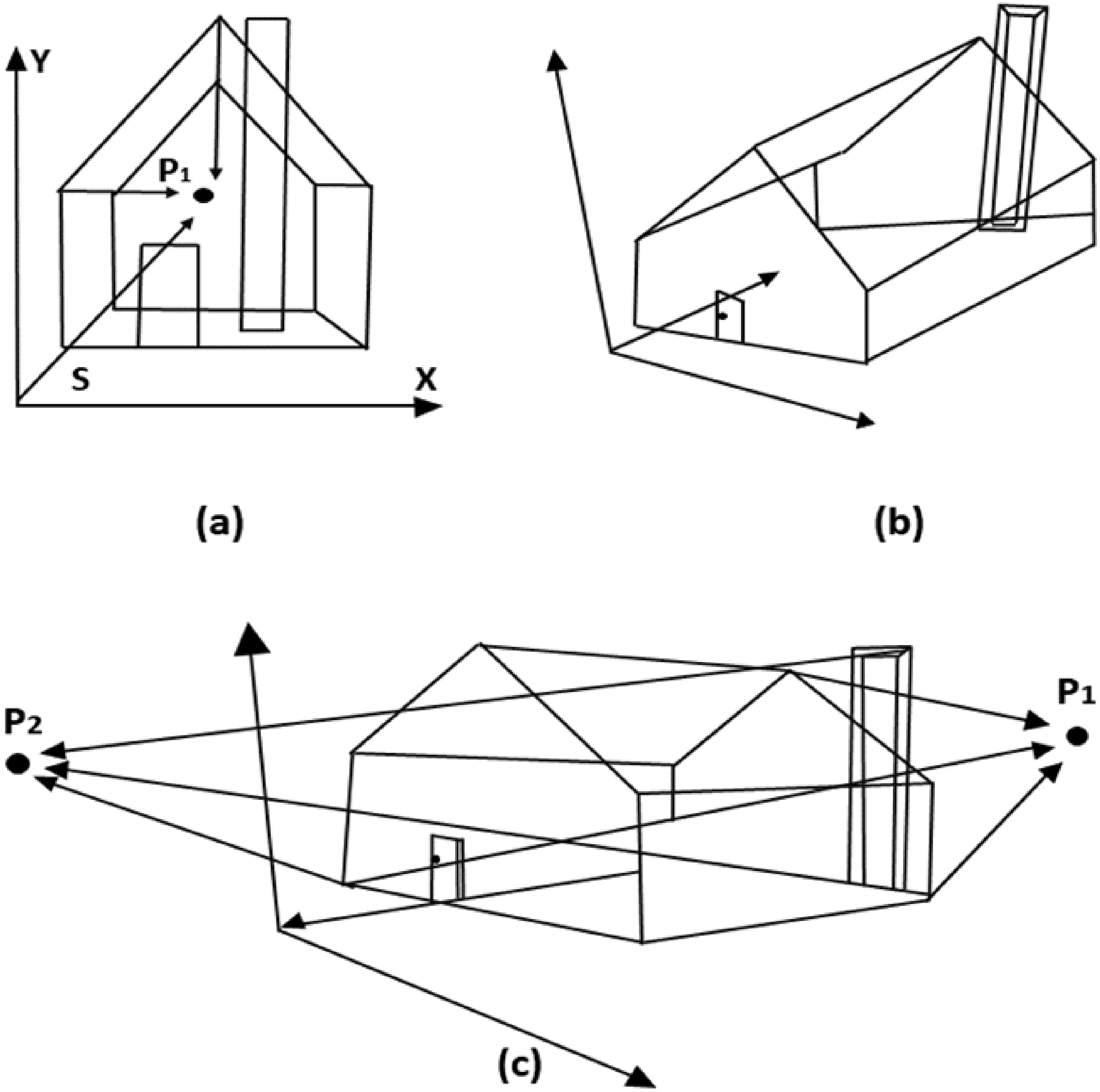


Fig: Prespective Projection

Anomalies in Perspective Projection

It introduces several anomalies due to these object shape and appearance gets affected.

1. **Perspective foreshortening:** The size of the object will be small of its distance from the center of projection increases.
2. **Vanishing Point:** All lines appear to meet at some point in the view plane.
3. **Distortion of Lines:** A range lies in front of the viewer to back of viewer is appearing to six rollers.

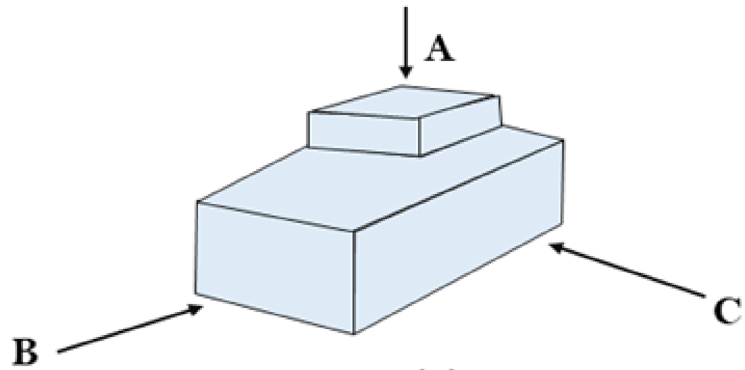


Foreshortening of the z-axis in fig (a) produces one vanishing point, P_1. Foreshortening the x and z-axis results in two vanishing points in fig (b). Adding a y-axis foreshortening in fig (c) adds vanishing point along the negative y-axis.

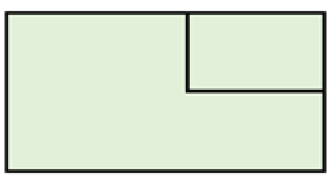
Parallel Projection

Parallel Projection use to display picture in its true shape and size. When projectors are perpendicular to view plane then is called **orthographic projection**. The parallel projection is formed by extending parallel lines from each vertex on the object until they intersect the plane of the screen. The point of intersection is the projection of vertex.

Parallel projections are used by architects and engineers for creating working drawing of the object, for complete representations require two or more views of an object using different planes.

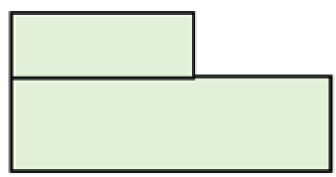


(a)



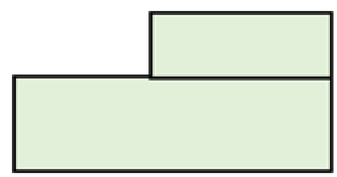
Parallel projection from top i.e. A direction

(b)



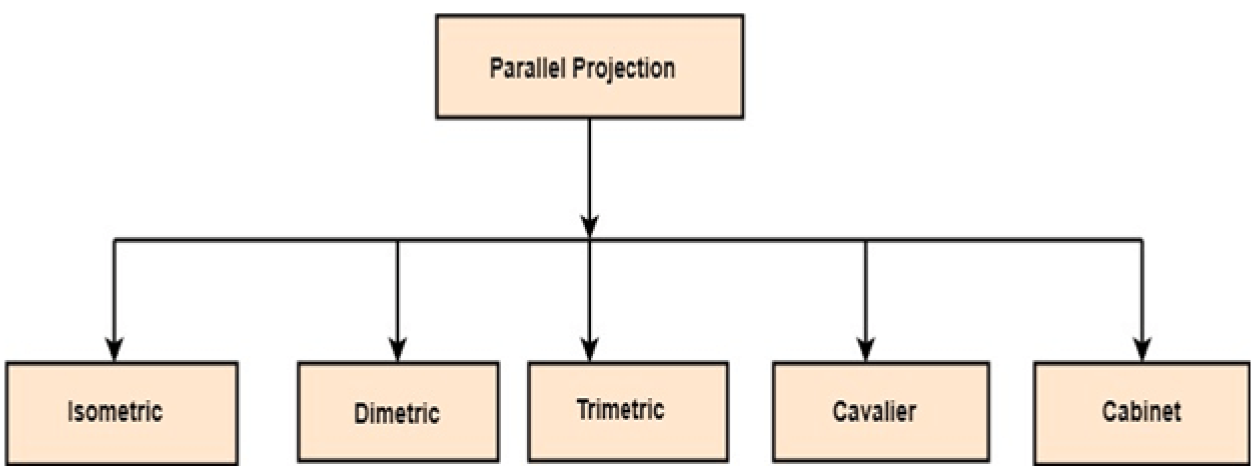
Parallel projection from top i.e. B direction

(c)

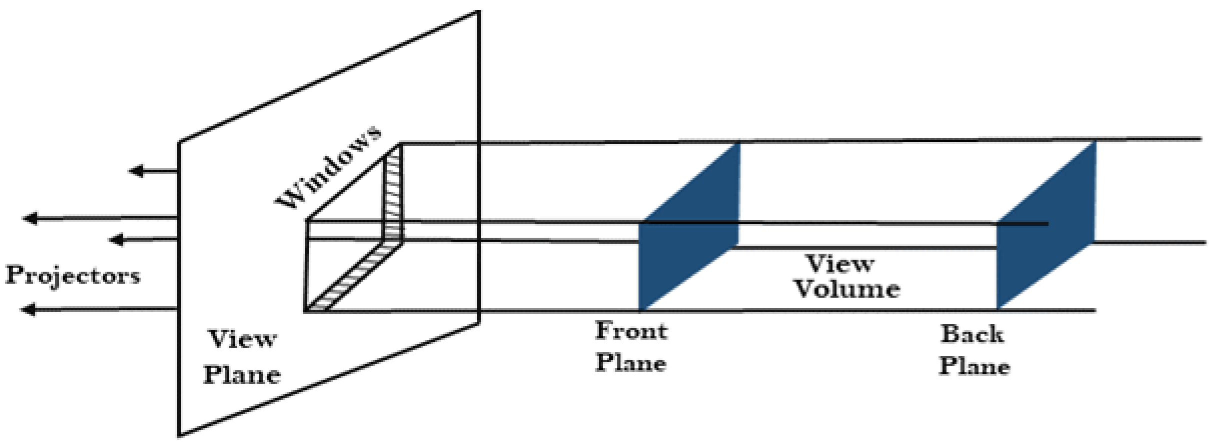


Parallel projection from top i.e. C direction

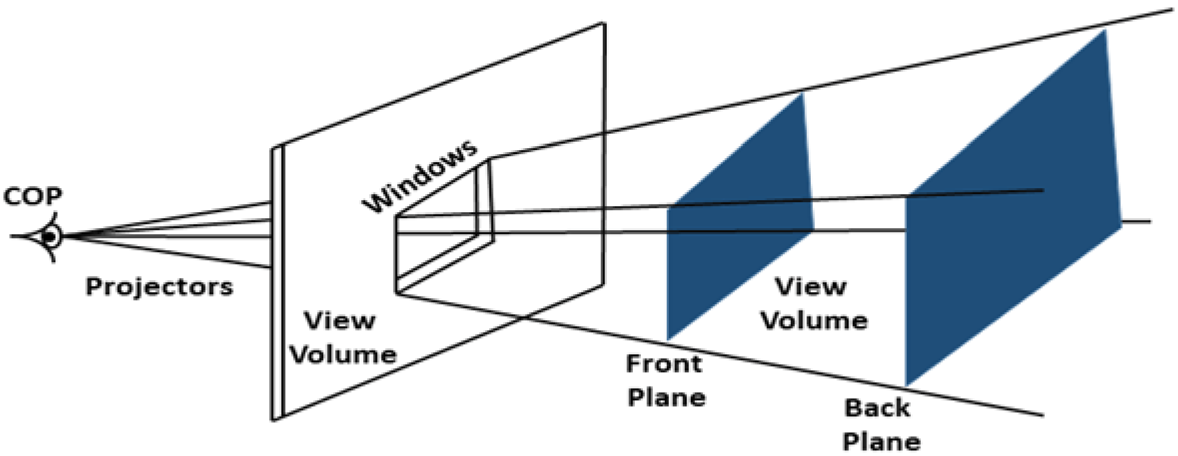
(d)



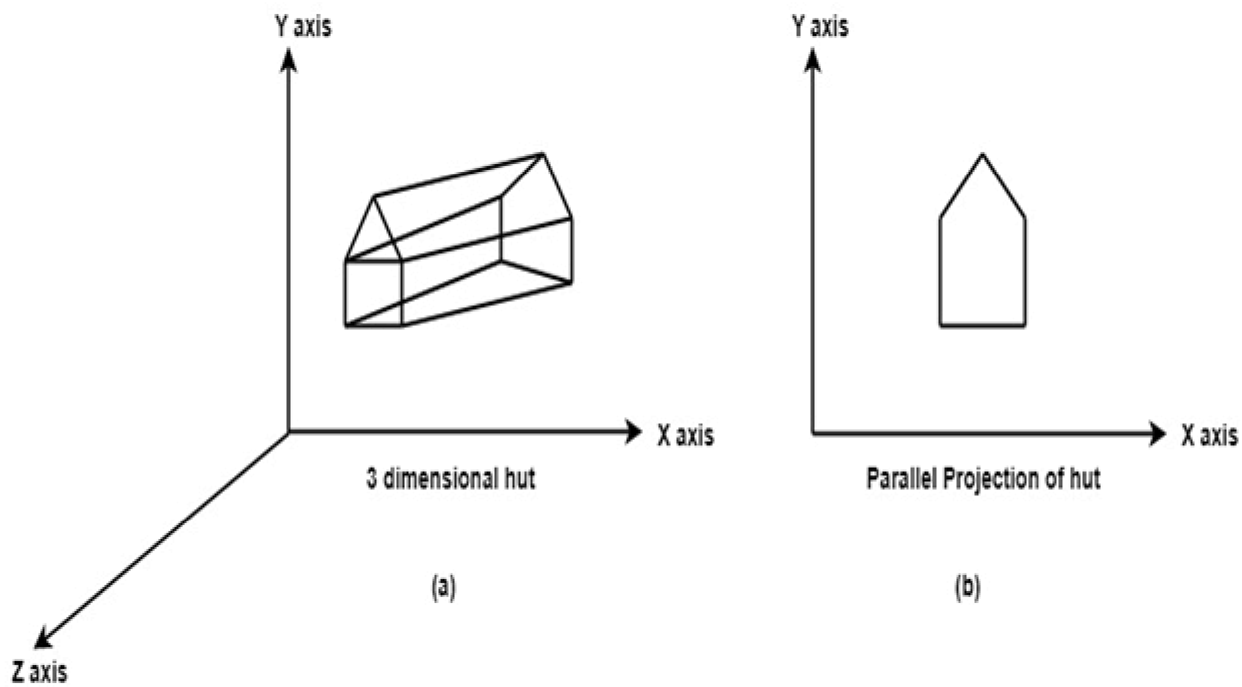
1. **Isometric Projection:** All projectors make equal angles generally angle is of 30°.
2. **Dimetric:** In these two projectors have equal angles. With respect to two principle axis.
3. **Trimetric:** The direction of projection makes unequal angle with their principle axis.
4. **Cavalier:** All lines perpendicular to the projection plane are projected with no change in length.
5. **Cabinet:** All lines perpendicular to the projection plane are projected to one half of their length. These give a realistic appearance of object.

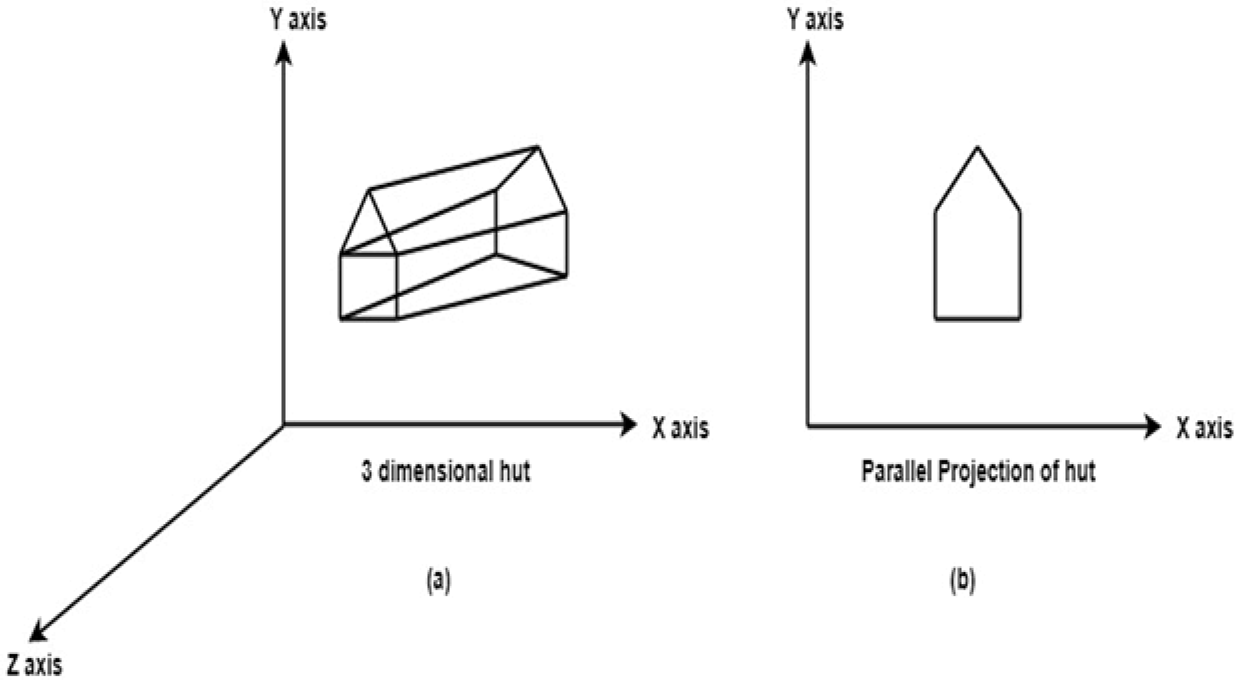


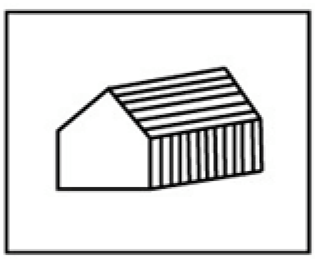
(a) Viewing Volume in orthographic projection



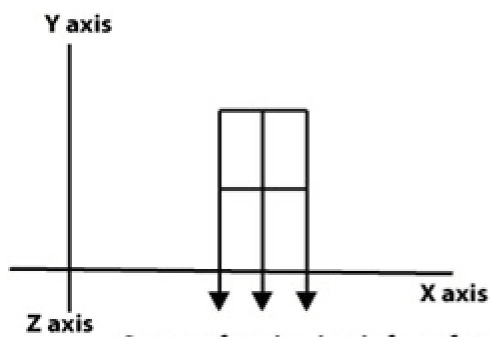
(b) Viewing volume in perspective projection



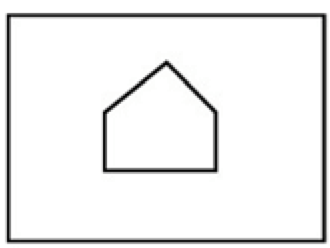




(a) **Object original**

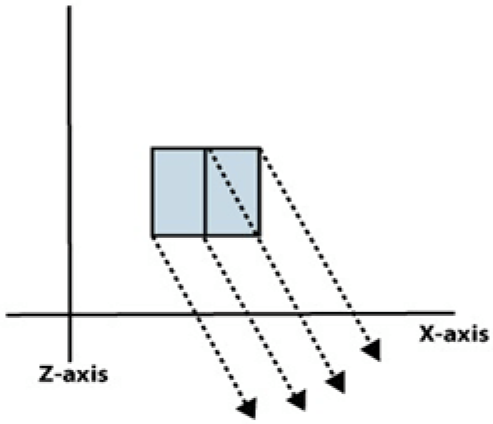


Center of projection is from front
(b)

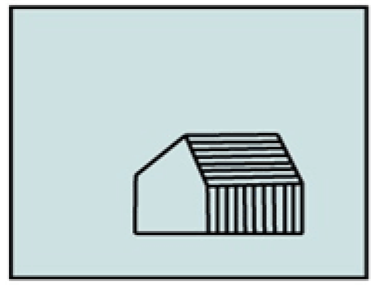


projected object
(c)

Fig (a) shows original object. Fig (b) shows object when projection is taken. Fig (c) gives projected object.



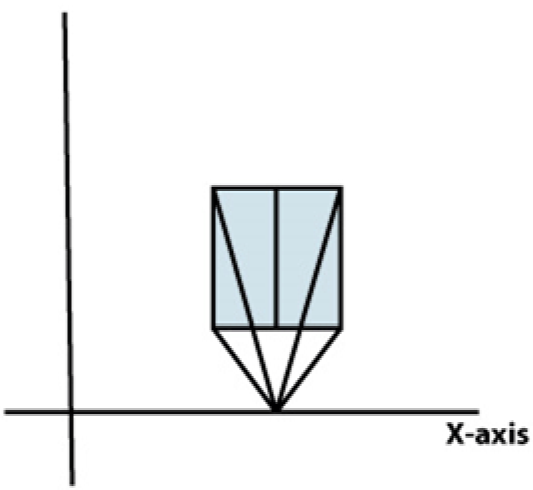
(d)



(e)

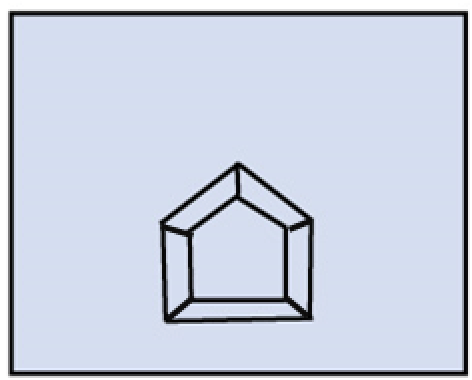
Fig (d) changes the direction of projection

Fig (e) shows object after changing direction of projection.

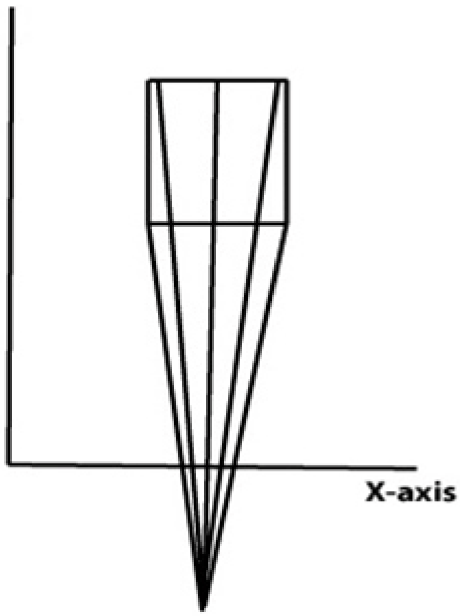


Center of projection meet at a chosen point

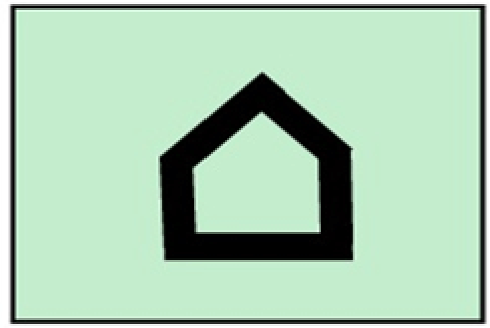
(a)



(b)



(a)



(b)